

TEACHING GEOMETRY WITH
The Geometer's Sketchpad

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Abstract

Technology has had and continues to have an influence on secondary and post-secondary education. Computers and calculators have changed the way mathematics is taught. This paper explores ways in which geometry can be taught using the dynamic geometry software *The Geometer's Sketchpad*. Three ways to utilize this software are presented. They include demonstration, investigation and discovery. Each method is explained through sample sketches which were created for this paper as well as future use in mathematics classrooms.

1 Introduction

Technology has become an instructional tool at both the secondary and collegiate levels. In 2000, the National Council of Teachers of Mathematics developed its "Technology Principle." This principle states that "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" [4]. According to Huetnick and Munshin, "student's engagement is generally heightened with the use of technology" [2]. One piece of computer software that enhances the teaching of mathematics is *The Geometer's Sketchpad*. There are several approaches to using *The Geometer's Sketchpad* in the classroom. This paper includes sketches that were created for use as demonstration, investigation and discovery.

2 *The Geometer's Sketchpad*

The Geometer's Sketchpad (*Sketchpad*) is a dynamic geometry software package that allows the user to construct and manipulate geometric objects. The user is able to construct anything that can be constructed using a straight edge and compass. *Sketchpad* follows the same rules as traditional constructions. For instance, in order to construct a perpendicular line using straightedge and compass, a line and a point on or off the line is needed first. So to be able construct a perpendicular line in *Sketchpad*, the user must first select a line and a point. By following the same rules as traditional constructions, *Sketchpad* "can reinforce geometric principles of construction encountered in traditional methods, but not fully internalized" [2].

In addition to constructions, the user is able to measure angles, the lengths of segments, and area to name a few. Another useful tool is the calculate function which allows the user to

select measurements for use in calculations. This way when an object is changed, *Sketchpad* automatically recalculates using the rule. All of these functions are easy to find and use in *Sketchpad* and were essential to all the sketches presented in this paper.

3 Demonstration

One approach to using *Sketchpad* in the classroom is for demonstration. The teacher or student can create a sketch and use it to explain a concept to the class. The sketch titled “Calculating Pi” is designed for this purpose. This sketch, as seen in Figure 1, is suitable for use in middle and high school classes. The main purpose of this sketch is to supplement a discussion of π . The user clicks the “Animate Objects” button and watches what happens. As the number of sides of the polygon increases, the polygon begins to resemble a circle and the estimation of π becomes more accurate. Figure 2 below shows the sketch during animation.

The concept for this sketch developed from a paper and pencil activity. The original activity asked students to construct inscribed and circumscribed regular polygons the areas of which were used to approximate π . Completing this activity with paper and pencil is time consuming. However, the results about an estimation of pi can fascinate students. Recreating this activity in a sketch using *Sketchpad* reduces the time it takes to reach the point of fascination. This sketch also shows off many features of *Sketchpad* beyond basic constructions and measurements.

While the mathematics behind this sketch is simple, creating it in *Sketchpad* presented some challenges. Many tools were used in the creation of this sketch. The first step was to construct a circle and its radius. Then *Sketchpad* was instructed to divide 360° by 4, the number of sides of a square. This determined the arc measure between vertices of the square. Then several line tools were employed to construct one side of an inscribed square and one side of a circumscribed square. The areas of the constructed sections of the inscribed and circumscribed squares were measured. The challenge was then to rotate these sides around the circle to create the entire inscribed and circumscribed squares. Since the second vertex of the inscribed square as well as the circumscribed side were drawn based on one point, the first vertex, the iterate function provided in *Sketchpad* could be used. The points and segments were iterated around the circle until the complete squares were made. The next important step was to add the areas of the sectors to find the area of each square. This was easy to do using the calculate tool. Using the calculate tool again, each area was divided by

Calculating Pi

Click on the animate object button. Note what happens to the values in the top right corner as number of sides of the polygons increase.

Animate Objects

$$2.00000 \leq \pi \leq 4.00000$$

Inner Area = 13.92 cm²

Outer Area = 27.85 cm²

r = 5.28 cm

Show Parameter, Table

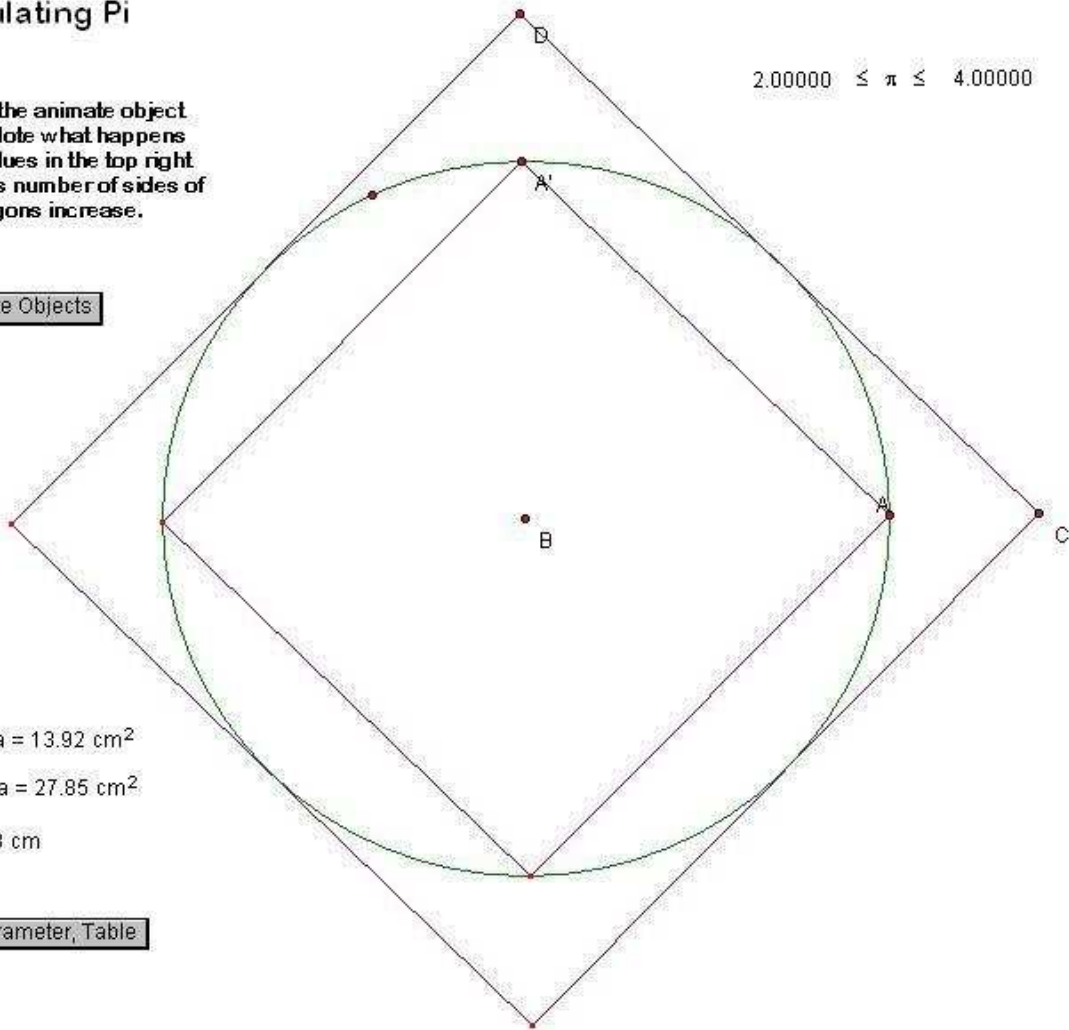


Figure 1: Estimating Pi Demonstration Sketch

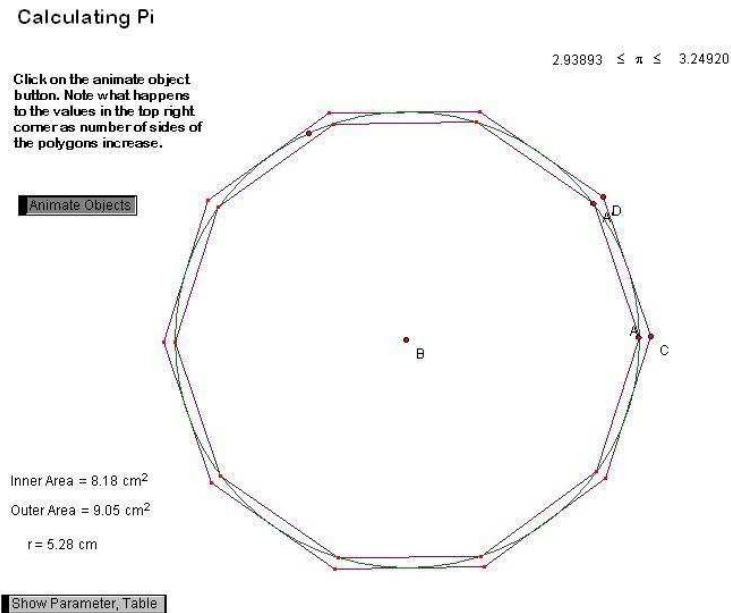


Figure 2: Estimating Pi Sketch During Animation

the radius of the circle squared, in order to find an estimate of π .

Although this sketch was nice, the square needed to be changed to a regular pentagon, and then to a hexagon and so forth to find a better estimate of π . At this point the parameter tool was discovered in *Sketchpad*. This parameter, n , would be the divisor of the degrees in a circle thus creating regular inscribed and circumscribed n -gons. Manually changing the parameter was effective but not user friendly. The “iterate to depth” option, hidden in *Sketchpad*, became a good solution. Now that the parameter was iterating discretely, regular polygons were created cleanly and quickly. An animate button was added so that the polygons would be created continuously at the click of the mouse. The sketch was then cleaned up by adding a hide/show button. When the user clicks on the hide/show button, the parameter along with the table of area calculations appears. This is necessary so that the user can reset the parameter. It is also helpful for explaining how the sketch is designed to act as it does.

During recent in-class instruction about polygons, students asked if a polygon could have a million sides. To help explain that a polygon could have a million sides but that actually creating one would be difficult, the sketch was altered slightly by hiding the circumscribed polygon. By animating only the inscribed polygon it was easier to see that when the polygon had about 30 sides it looked just like a circle. While this fascinated the class, the animation

Calculating the Area of a Triangle

$a = 10.69 \text{ cm}$
 $b = 9.59 \text{ cm}$
 $c = 8.30 \text{ cm}$

METHOD 1 The most common formula for the area
 A of a triangle is $A = \frac{1}{2}bh$,
 where b is the length of the base and h is the height.
 $h = 7.12 \text{ cm}$
 $b = 10.69 \text{ cm}$
Show height Area 1 = 38.05 cm^2

METHOD 2 $A = \sqrt{s(s-a)(s-b)(s-c)}$
 This involves the semiperimeter
 $s = \frac{(a+b+c)}{2}$
 $s = 14.29 \text{ cm}$ Area 2 = 38.05 cm^2

METHOD 3 $A = sr$
 This way of calculating area uses the semiperimeter s and the radius r of the inscribed circle.
Show Inscribed Circle Area 3 = 38.05 cm^2
Click for a pictorial hint.

METHOD 4 $A = \frac{abc}{4R}$
 This method involves the radius of the circumscribed circle.
Show Circumscribed Circle Area 4 = 38.05 cm^2
Click for a pictorial hint.

Prove that the area of a triangle is given by $A = s \cdot r$.

Prove that the area of a triangle is given by $A = \frac{abc}{4R}$.

Figure 3: Area of a Triangle Sketch

itself had a wow factor.

4 Investigation

Another way *Sketchpad* can be utilized is for investigation. In this case, investigation means to play with concepts already presented. A teacher could lead a whole class through an investigation or students could work independently or in groups. The “Area of a Triangle” sketch was created for investigation. It can be used in middle school, high school and college classrooms. The first two methods for finding area seen in this sketch are taught in middle and high school geometry. The second two formulas are not typically seen in secondary education but may be included in a college geometry course. These last two methods bring together several concepts that students are exposed to in high school geometry. Therefore, this is a great enrichment activity for high school geometry students, especially those with advanced mathematical ability.

The sketch is designed to show that although the formulas may be very different, each

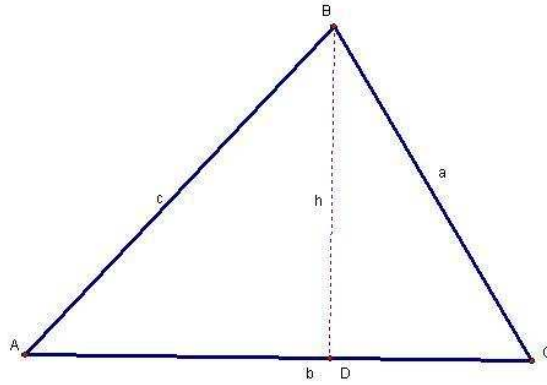


Figure 4: Triangle with Height

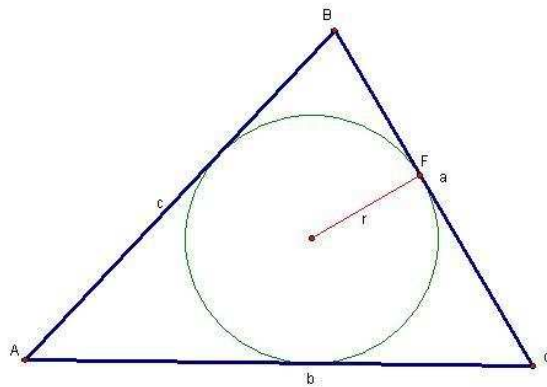


Figure 5: Inscribed Circle

one gives the area of a triangle. Throughout the sketch, students are able to move the points of the triangle to create new triangles, while observing the same properties. This allows the student to make visual sense of the method but at the same time understand that the method works for any triangle, not just the triangle originally constructed.

The first activity contains a hide/show button which makes the altitude of the triangle appear, as seen in Figure 4. It also shows the height and base measurements, which *Sketchpad* uses to calculate the area using the well known formula “one-half base times height.” This is labeled Area 1. The second activity shows the formula for area using the semiperimeter. This calculation is labeled Area 2. Students see that although this second formula is very different from the first, it still finds the area of the triangle.

The third method for finding area of a triangle included in this sketch uses the radius of the inscribed circle along with the semiperimeter, as seen in Figure 5. This area calculation

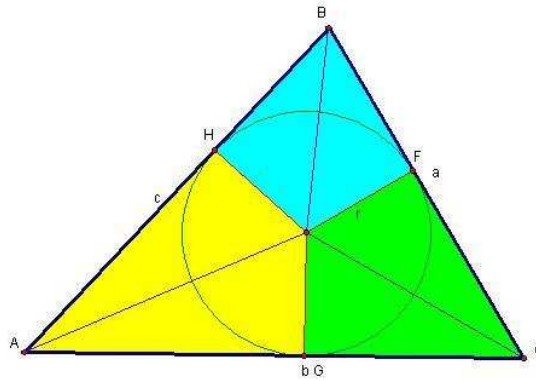


Figure 6: Inscribed Circle with Pictorial Hint

is labeled Area 3. By clicking on the “Show Inscribed Circle” button, the inscribed circle and its radius can be seen on the sketch. This part of the sketch also asks the student to prove that the formula does in fact give the area of a triangle. In order for the student to use this sketch independently, a hide/show button, which gives a pictorial hint, was included. Clicking the pictorial hint button makes line segments and shading appear. (See Figure 6.) These line segments and shaded areas are the main ideas for the proof. Once the student has a visual idea of how the formula can be proved, an attempt at a rigorous proof can be made.

The final activity on this sketch provides a formula for the area of a triangle which uses the radius of the circumscribed circle. This is labeled Area 4. Similar to the step above, the student can click to view the circumscribed circle and its radius, as seen in Figure 7. Once again the student is asked to prove that the area of a triangle is given by the formula. Just as in the previous activity, students are able to click for a pictorial hint. As can be seen in Figure 8, several lines appear. These lines are necessary for the proof. While the hints are designed to increase student independence, high school students, who are less familiar with proofs, may still need direction and help from the teacher when it comes time to write the proofs.

5 Discovery

“Computers provide instant calculations and rapidly generate graphics with which students can make and test conjectures” [1]. *Sketchpad* can be a tool for students of all ages to test conjectures and discover concepts. By discovering a concept themselves, students construct

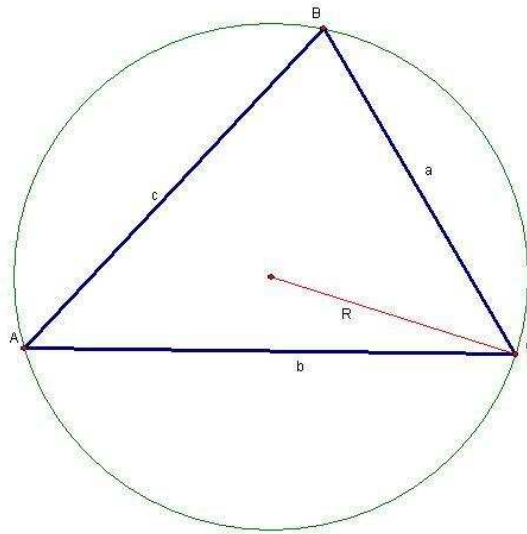


Figure 7: Circumscribed Circle

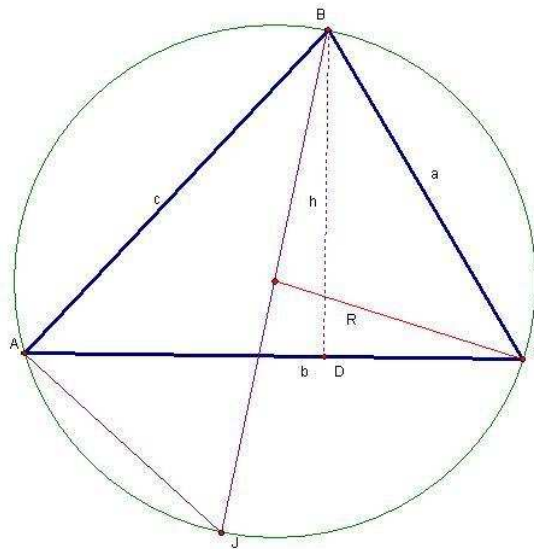
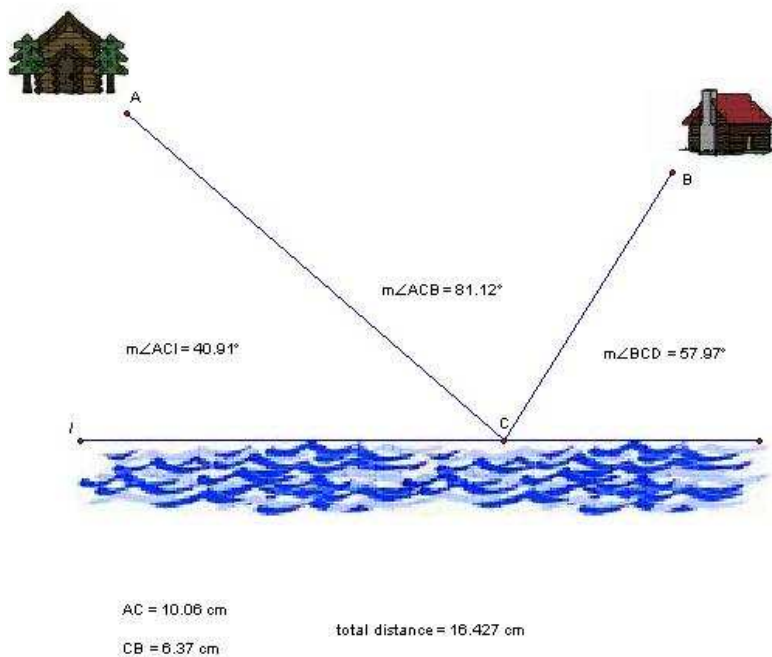


Figure 8: Circumscribed Circle with Pictoral Hint

DISCOVER HERON'S THEOREM

THE PROBLEM:

Kerri is going to visit her friend Erin. Kerri lives in Cabin A and Erin lives in Cabin B. Erin asks Kerri to bring water from the stream. What path should Kerri take so that she walks the shortest distance?



DIRECTIONS

1. Move point C along the line to find where the distance is minimized. Move points A and B and try again. Under what conditions is the length of the path minimized? Make a conjecture.

2. Click **Hide Pictures**

3. Try to prove your conjecture.

4. Click **Show Theorem**

5. Construct the segments in Heron's Theorem.

Show Segments

6. Using this new information, try again to prove your conjecture.

Show Hint

Figure 9: Discover Heron's Theorem Sketch

and internalize information in their own way and create their own understanding. When students have ownership of their learning, the amount of learning that occurs is increased. Thus, using *Sketchpad* as a tool for discovery can really enhance the learning that takes place in a mathematics classroom.

The sketch titled "Discover Heron's Theorem" is designed for student discovery. Students are presented with a problem and accompanying diagram. Although Heron's Theorem is not typically taught in a high school geometry class, it is a great extension activity after students have learned about reflections. In order for students to discover the result, they are required to recall and apply previously learned knowledge about reflections, angles and triangles. This sketch provides a lot of direction for moving points as well as providing the measures of important segments and angles. Students who are familiar with *Sketchpad* could be presented with the problem and a blank sketch. The students could then create their own sketch to solve the problem and discover the result.

By providing more explicit instructions, lower level students can more easily and quickly move through the discovery process. Pre-constructed segments and measurements along

with detailed directions put the emphasis on discovering the result and not on the student's ability to use *Sketchpad*. These components also reduce dependency on the teacher and thus accelerate the process of discovery for each student and the class as a whole.

Now let us walk through the sketch. The sketch poses the following problem: "Kerri is going to visit her friend Erin. Kerri lives in Cabin A and Erin lives in Cabin B. Erin asks Kerri to bring water from the stream. What path should Kerri take so that she walks the shortest distance?" After the problem is presented, the first question asks the student to move point C and look at the lengths of the segments in order to find the shortest distance from Cabin A to the river and then to Cabin B. The next step is for students to make a conjecture about the conditions under which the distance is minimized.

Next, students hide the pictures in the sketch so that only the segments remain. This symbolizes the move from the "realistic" problem solving aspect of the problem to the mathematical model. Now that students are focused on the mathematics, they attempt to prove the conjecture they make. There is a potential for students to get hung up on this step. Depending on the ability level of the student, this step may be skipped or a time limit for attempting a proof may be set. A time limit for attempting a proof may also be useful for students who would otherwise rush through this step.

In step 4, the student clicks a button to reveal the following: "Heron's theorem says: For two points on the same side of a line, the shortest path from the first point to the line and then to the second point is by way of the point of intersection of the line and the segment from the first point to the reflection of the second point." While a geometric theorem written out in words may be beneficial for some students, many students need and immediately look for a picture to demonstrate what the words are explaining. Step 5 of this sketch provides the needed visual. By clicking on "Show Segments," the line segments mentioned in Heron's Theorem appear. If the student has left Point C at the place where the path is minimized, s/he will notice that the segments that appear overlap with part of the path. This may invoke reactions of satisfaction for being correct or disappointment for being "wrong."

Finally, in the last step, the student is able to use the sketch to prove the conjecture that was made at the start of the sketch. As in the previous sketch, a hint is built into the sketch through the use of a hide/show button. The sketch at this step can be seen in Figure 10. With the inclusion of these hints, students can troubleshoot on their own and not have to rely on the teacher for help. If this theorem is discussed in the textbook for the class, a further hide/show hint button could be added which directs students to the appropriate page or section of the textbook. There is also the option of linking to a webpage for further hints

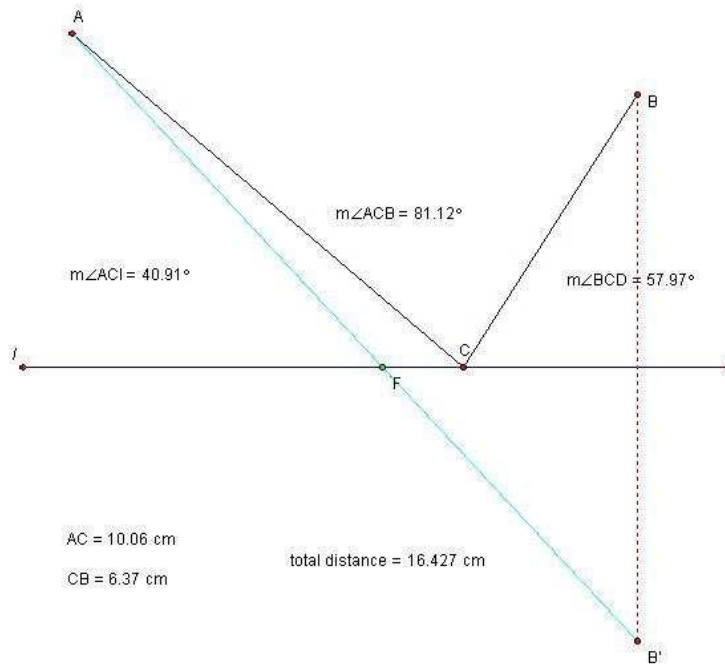


Figure 10: Proving Heron's Theorem

or explanation. The addition of a hint makes this sketch an independent activity. Since the teacher is not required for successful use of the sketch, the student is able to take ownership of the learning.

6 Conclusion

The Geometer's Sketchpad can be used in many ways in a classroom setting. A tool for demonstration and investigation, *Sketchpad* has features that enhance the teaching of mathematics. More importantly, *Sketchpad* allows students to be in control of their learning and actively construct their own mathematical understandings. Technology, such as *Sketchpad*, is a great tool for teachers and students in secondary and post-secondary education.

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